



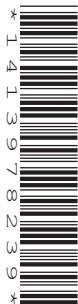
Oxford Cambridge and RSA

Wednesday 22 May 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Time allowed: 2 hours 40 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **144**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Section A (36 marks)

- 1 By expressing $\frac{1}{r+1} - \frac{1}{r+2}$ as a single fraction, find $\sum_{r=1}^n \frac{1}{(r+1)(r+2)}$ in terms of n . [4]
- 2 Two complex numbers are given by $u = -1 + i$ and $v = -2 - i$.
- (a) (i) Find $u - v$ in the form $a + bi$, where a and b are real. [1]
- (ii) **In this question you must show detailed reasoning.**
- Find $\frac{u}{v}$ in the form $a + bi$, where a and b are real. [3]
- (b) Express u in exact modulus-argument form. [3]
- 3 The equation $2x^3 - 2x^2 + 8x - 15 = 0$ has roots α , β and γ .
Determine the value of $\alpha^2 + \beta^2 + \gamma^2$. [4]
- 4 The equation of a curve is $y = \frac{1}{\sqrt{k^2 + x^2}}$, where k is a positive constant. The region between the x -axis, the y -axis and the line $x = k$ is rotated through 2π radians about the x -axis.
Given that the volume of the solid of revolution formed is 1 unit^3 , find the exact value of k . [4]
- 5 (a) Given that $\mathbf{u} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{u} \times \mathbf{v} = \begin{pmatrix} 1 \\ b \\ 3 \end{pmatrix}$, find a and b . [3]
- (b) Using $\mathbf{u} \times \mathbf{v}$, determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$, given that this angle is acute. [3]

6 On separate Argand diagrams, sketch the set of points represented by each of the following.

(a) $|z - 1 - 2i| \leq 4$. [3]

(b) $\arg(z + i) = \frac{1}{3}\pi$. [3]

7 (a) Explain why $\int_1^2 \frac{1}{\sqrt[3]{x-2}} dx$ is an improper integral. [1]

(b) **In this question you must show detailed reasoning.**

Use an appropriate limit argument to evaluate this integral. [4]

Section B (108 marks)

- 8 (a) Specify fully the transformation T of the plane associated with the matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$ and λ is a non-zero constant. [2]
- (b) (i) Find $\det \mathbf{M}$. [1]
- (ii) Deduce **two** properties of the transformation T from the value of $\det \mathbf{M}$. [2]
- (c) Prove that $\mathbf{M}^n = \begin{pmatrix} 1 & n\lambda \\ 0 & 1 \end{pmatrix}$, where n is a positive integer. [4]
- (d) Hence specify fully a **single** transformation which is equivalent to n applications of the transformation T . [1]
- 9 A curve has polar equation $r = a \sin 3\theta$, for $0 \leq \theta \leq \pi$, where a is a positive constant.
- (a) Sketch the curve. Indicate the parts of the curve where r is negative by using a broken line. [3]
- (b) **In this question you must show detailed reasoning.**
- Determine the area of one of the loops of the curve. [5]
- 10 (a) Write down the first three terms of the Maclaurin series for $\ln(1+x^3)$. [1]
- (b) Use these three terms to show that $\ln(1.125) \approx \frac{n}{1536}$, where n is an integer to be determined. [3]
- (c) Charlie uses the same first three terms of the series to approximate $\ln 9$ and gets an answer of 147, correct to 3 significant figures. However, $\ln 9 = 2.20$ correct to 3 significant figures.
- Explain Charlie's error. [2]

- 11** The plane Π has equation $2x - y + 2z = 4$. The point P has coordinates (8, 4, 5).

(a) Calculate the shortest distance from P to Π . [2]

The line L has equation $\frac{x-2}{3} = \frac{y}{2} = \frac{z+3}{4}$.

(b) Verify that P lies on L. [2]

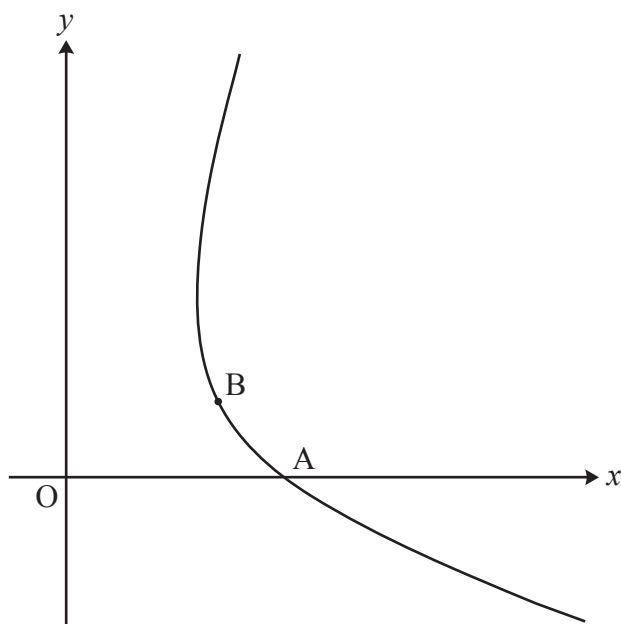
(c) Find the coordinates of the point of intersection of L and Π . [3]

(d) Determine the acute angle between L and Π . [4]

(e) Use the results of parts **(b)**, **(c)** and **(d)** to verify your answer to part **(a)**. [3]

- 12** The diagram shows the curve with parametric equations

$$x = 2 \cosh t + \sinh t, \quad y = \cosh t - 2 \sinh t.$$



(a) The curve crosses the positive x -axis at A.

(i) Determine the value of the parameter t at A, giving your answer in logarithmic form. [4]

(ii) Find the x -coordinate of A, giving your answer correct to **3** significant figures. [2]

(b) The point B has parameter $t = 0$.

Determine the equation of the tangent to the curve at B. [6]

- 13** The complex number z is defined as $z = \frac{1}{3}e^{i\theta}$ where $0 < \theta < \frac{1}{2}\pi$.

On an Argand diagram, the point O represents the complex number 0, and the points $P_1, P_2, P_3, \dots$ represent the complex numbers $z, z^2, z^3, \dots$ respectively.

(a) Write down each of the following.

(i) The ratio of the lengths $OP_{n+1} : OP_n$ [1]

(ii) The angle $P_{n+1}OP_n$ [1]

(b) (i) Show that $(3 - e^{i\theta})(3 - e^{-i\theta}) = a + b \cos \theta$, where a and b are integers to be determined. [2]

(ii) By considering the sum to infinity of the series $z + z^2 + z^3 + \dots$, show that

$$\frac{1}{3} \sin \theta + \frac{1}{9} \sin 2\theta + \frac{1}{27} \sin 3\theta + \dots = \frac{3 \sin \theta}{10 - 6 \cos \theta}. \quad [6]$$

- 14 (a)** Find the general solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 12e^{-x}$. [7]

You are given that y tends to zero as x tends to infinity, and that $\frac{dy}{dx} = 0$ when $x = 0$.

(b) Find the exact value of x for which $y = 0$. [5]

- 15** Three planes have equations

$$\begin{aligned} x + ky + 3z &= 1, \\ 3x + 4y + 2z &= 3, \\ x + 3y - z &= -k, \end{aligned}$$

where k is a constant.

(a) Show that the planes meet at a point except for one value of k , which should be determined. [4]

(b) Show that, when the planes do meet at a point, the y -coordinate of this point is independent of k . [6]

16 In this question you must show detailed reasoning.

Show that $\int_0^1 \frac{1}{\sqrt{x^2+x+1}} dx = \ln\left(\frac{a+b\sqrt{3}}{c}\right)$, where a , b and c are integers to be determined. [6]

- 17** In an industrial process, a container initially contains 1000 litres of liquid. Liquid is drawn from the bottom of the container at a rate of 5 litres per minute. At the same time, salt is added to the top of the container at a constant rate of 10 grams per minute. After t minutes the mass of salt in the container is x grams, and you are given that $x = 0$ when $t = 0$.

In modelling the situation, it is assumed that the salt dissolves instantly and uniformly in the liquid, and that adding the salt does not change the volume of the liquid.

- (a) (i) Show that the concentration of salt in the liquid after t minutes is $\frac{x}{1000-5t}$ grams per litre. [1]

- (ii) Hence show that the mass of salt in the container is given by the differential equation

$$\frac{dx}{dt} + \frac{x}{200-t} = 10. \quad [3]$$

- (b) Show by integration that $x = 10(200-t)\ln\left(\frac{200}{200-t}\right)$. [8]

- (c) (i) Hence determine the mass of salt in the container when half the liquid is drawn off. [2]

- (ii) Determine also the time at which the mass of salt in the container is greatest. [5]

- (d) When the process is run, it is found that the concentration of salt over time is higher than predicted by the model.

Suggest a reason for this. [1]

END OF QUESTION PAPER

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